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A Markov Process Model for Neuron Behavior in the Interspike Interval

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Our laboratory, like many others, has been investigating the origin of variability in the time between spike occurances in a repetitively firing neuron whose average spike frequency is constant. We have concentrated primarily on the relatively well studied spinal motoneuron, and have employed intracellular recording in an effort to observe directly those neural processes we feel are responsible for the variability. In summary, we have found that a model consisting of synaptic noise together with a deterministic spike generation process can adequately account for the interspike interval variability in a restricted--but commonly observed--class of neurons.

If a silent spinal motoneuron is impaled, one typically observes apparently random fluctuations in membrane potential. These fluctuations are believed to arise from the temporal summation of randomly occuring PSPs, and are thus termed "synaptic noise." Consideration of the way in which synaptic noise is produced suggests that an appropriate idealized description of the process is a strictly random current applied to a RC circuit. Letting V(t) represent the deviations of membrane potential from its mean level, the process we are considering may be written as the stochastic differential equation

$$dV(t) + \frac{1}{RC} V(t)dt = \sqrt{\frac{2}{RC}}\,\sigma dB(t)$$

Here RC is the lumped RC of the neuron, σ is a constant characterizing the size of the underlying conductance changes, and dB(t) is a random process representing the synaptic current; B(t) is a Weiner-Einstein process. It is well known that V(t) is the Uhlenbeck-Ornstein (U-O) process. According to this model, synaptic noise should be normally distributed with a covariance

$$E[V(t)V(\tau)] = \sigma^2 \exp\left(\frac{-|t-\tau|}{RC}\right)$$

This is approximately true for many neurons.

In most spinal motoneurons we encounter, the membrane repolarizes at the end of a spike, and then increases more or less ramp-like to the next firing level. Descriptively, then, the deterministic spike generating process may be represented by

$$y(t) = ct + s$$

where y(t) is the membrane potential, c and s constants; the firing level has been taken as zero.

Experimental observations and theoretical considerations both suggest that, at least near the firing level, synaptic noise simply adds to the ramp arising from the spike generation mechanism. A stochastic model of the neuron's behavior in the interspike interval then is

$$G(t) = ct + s + V(t)$$

It can be shown that the transition probabilities characterizing this process satisfy the equation

$$\frac{\partial p}{\partial t} = \frac{\sigma^2}{RC}\frac{\partial^2 p}{\partial G^2} - \left(c - \frac{\sigma^2 - ct}{RC}\right)\frac{\partial p}{\partial G} + \frac{1}{RC}\frac{\partial}{\partial G}(Gp) \quad (1)$$

It is important to note that all constants in this equation can be estimated from observations made with an intracellular microelectrode.

To predict the interspike interval histogram, it is necessary to solve equation (1) under the condition that the transition probability vanish at zero (the firing level). Since we have experienced difficulty in obtaining an exact solution, we have resorted to Monte Carlo solutions. Figure 1 compares, on a normal probability scale, the observed and Monte Carlo-predicted cumulated interspike interval histograms for two different mean frequencies of the same motoneuron. All constants were obtained directly from intracellular recordings; that is, no free parameters were estimated from the histogram we wished to predict. The observed and predicted distributions are not significantly different according to the Kolomogorov-Smirnov goodness-of-fit test.

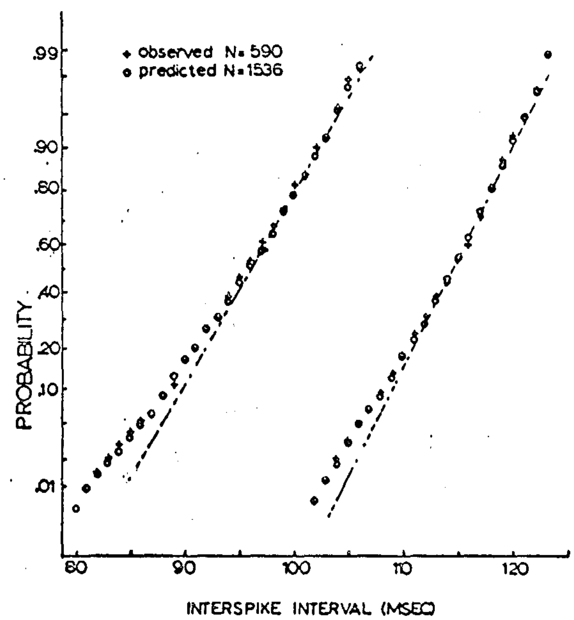

Figure 1